A MAD-X Primer

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Abstract

The purpose of this note is to serve as an introduction to MAD-X for the novice user. It describes the basic building elements and the most important commands to define a machine and perform the most important calculations. It cannot replace a reference manual for the more advanced and demanding use of MAD-X, but should help starting up the design of a new accelerator lattice and to understand existing MAD-X input. This note is a writeup of a presentation given at a course on optics design at the CERN Accelerator School at DESY Zeuthen, 2003.
1 Introduction

The MAD-X (Methodical Accelerator Design) program is a general purpose accelerator and lattice design program. The main aim of this note is to allow the beginner to understand the main building blocks of MAD-X and to set up and use a basic machine. The most important features are described and discussed. Details and more advanced options must be left to the MAD-X reference manual [1, 2].

This note is a writeup of a presentation given at a course on optics design at the CERN accelerator school [3].

The main objectives of an accelerator design program are:

- Read the elements and their sequence from a file.
- Calculate the optics parameters from a machine description.
- Define and compute (match) the desired properties of such a machine.
- Simulate and correct possible machine imperfections.
- Simulate the beam dynamics in the designed machine.

Both, circular and linear accelerators (or beam lines) are normally handled by such programs.

1.1 What is needed as input to design an accelerator lattice?

The design of a machine requires the definition of the machine and the basic ingredients are:

- Definition of the properties of all machine elements.
- Strength of all active machine elements.
- Position of all machine elements in the machine, i.e. the order in which they appear in the accelerator or beam line.

These definitions are helped by a well adapted and defined input language.

2 MAD-X language

For the definition of a machine and the execution of actions, an input language is used following a well defined syntax and grammar.

2.1 Coordinate system

A circular or linear machine in MAD-X is a sequence of elements placed along the reference orbit which is defined as the path through the ideal magnets of a charged particle with the reference momentum (Fig. 1). This defines the coordinate system \((x, y, s)\) where \(x\) is the transverse coordinate in the bending plane (usually horizontal, positive values to the outside) and \(y\) is the orthogonal transverse coordinate (usually vertical, positive values upward). The local variable \(s\) is the tangent to the reference orbit. The coordinate system is therefore a local system moving along the reference path.

2.2 Conventions

Units:
For all computations MAD-X uses SI units, i.e. in particular metre and radian for length and angle.

Input:
The MAD-X input consists of a sequence of statements (commands, actions, or declarations etc.). All statements are free format with "," as separators, can occupy any number of lines and are terminated by a semicolon ";".
Figure 1: Local coordinate system as used by MAD-X.

Blank lines have no effect. All statements are not case sensitive with the exception of strings enclosed in "".

Commands or statements can be given a label which allows them to be used or re-used later on:
LAB01: mycommand;

Comments:
A single line (not a statement across several lines !) can be commented by "!" or "//". A region of lines can be commented by enclosing them between "/*" and "*/".

2.3 Variables and expressions

Parameter values:
Integer or floating point numbers can be assigned to named parameters and can be used in further declarations or commands, e.g.:
LENDIP = 8.0;
Various names (keywords used by MAD-X) are protected and cannot be used as variables or labels. The numerical value of an assignment can be replaced by an expression, e.g.:
LNFX = 6912.00/(NCELL*(4*LBEND + 2*LQUAD));

Expressions:
Parameters and variables can be used in expressions, in particular to define dependent quantities. Standard arithmetic operations and functions such as SQRT(), EXP(), trigonometric functions etc. can be used in the expressions as well as random number generators [1, 2].
E.g.: ANGLE = 2.0*PI/NBEND;
can compute the bending angle of a dipole, given the total number of bending magnets ($nbend$). The constant $PI$ is predefined in MAD-X, together with many other important constants and particle properties.
Deferred expressions:
The usual expressions are evaluated once when the parameter is used,
e.g. DX = GAUSS()*0.001;
assigns a random number following a Gaussian distribution with a width of 1 mm to the parameter DX.
This value is kept in all computations.
A deferred expression is declared by "=" instead of "=" and is evaluated every time the parameter is
used, e.g.:
DX := GAUSS()*0.001;
assigns a different random number everytime the parameter DX is used in the program
The distinction between normal and deferred expressions becomes important for error assignment
and matching.

3 Machine description

3.1 Thick and thin elements

For the calculations, the elements can be defined either as so-called thick lenses with a finite length or
as so-called thin lenses with zero length. In the latter case, the effects of an element (e.g. a magnet) on
the beam are represented as impulses (kicks) at a fixed value s on the reference orbit. This simplifies
the treatment since it allows to treat the machine as a series of linear transformations separated by the
"kicks" at the positions of the thin elements. This method is very fast and symplectic by construction
and it is therefore best suited for particle tracking.

The disadvantage is the loss of precision when the magnets are very long (compared to the size of
the machine) or when fringe fields are important. Part of this precision can be recovered by sub-dividing
the magnets into slices, i.e shorter sections, each representing a thin lens.

3.2 Element definition

Elements are defined using the concept of element classes. All quadrupoles in a machine belong to the
class QUADRUPOLE. We can define subclasses with different properties with statements like:

MQL: QUADRUPOLE, L=5.0;
MQS: QUADRUPOLE, L=1.5;

where we define two classes (MQL and MQS) of quadrupoles of different length (thick elements).
The definitions can be used to define the real quadrupoles like e.g.:

QFL01: MQL; // Focusing quadrupoles
QFS01: MQS;
QDL01: MQL; // Defocusing quadrupoles
QDS01: MQS;

The quadrupoles defined like this inherit all properties of the class unless they are specified explicitly,
in which case they are overwritten. All numerical attributes in a class definition can be expressions.

Dipole magnets can be defined as rectangular (RBEND) or sector (SBEND) bending magnets. E.g.:

MBL: RBEND, L=14.3;
MBS: RBEND, L=5.0;
The length of a rectangular bending magnet is by default the arc length. All details on the definition of bending magnets are found in the reference manual [1, 2].

3.3 Element strength definition

3.3.1 Dipoles

The strength of a bending magnet is specified by the bending angle or alternatively the dipole coefficient $k_0$:

$$k_0 = \frac{1}{\rho/c} B_y \quad \text{[in T]} = \frac{1}{\rho} \frac{angle}{l} \quad \text{[in rad/m]}$$

In the latter case, a finite length must be specified.

The definition for a dipole magnet is:

MB001: RBEND, L=14.3, ANGLE=2*PI/1132; // Total number of dipoles is 1132

or, alternatively:

MB001: MBL, ANGLE=2*PI/1132; // Total number of dipoles is 1132

using the defined sub-class.

3.3.2 Quadrupoles

We define a quadrupole by its quadrupole coefficient $k_1$:

which is defined as:

$$k_1 = \frac{1}{\rho/c} \frac{\delta B_y}{\delta x} \quad \text{[in T/m]} = \frac{1}{l \cdot f}$$

We define quadrupoles as:

QF007: QUADRUPOLE, L=5.0, K1 = +0.00147235;
QD007: QUADRUPOLE, L=5.0, K1 = -0.00147235;

or using sub-classes:

QF007: QFL, K1 = +0.00147235;
QD007: QDL, K1 = -0.00147235;

3.3.3 Sextupoles

Higher order multipoles such as sextupoles we can define as:

SF007: SEXTUPOLE, L=1.4, K2 = +0.00147235;

with:

$$k_2 = \frac{1}{\rho/c} \frac{\delta^2 B_y}{\delta^2 x} \quad \text{[in T/m^2]}$$
3.3.4 Orbit correctors

Orbit correction dipoles are identified by the keyword KICKER. The strength of an orbit corrector is the deflection angle (KICK) measured in rad. Valid definitions are:

LKICK = 0.1;
MCV01: VKICKER, L=0.1, KICK:=KCV01;
MCV02: VKICKER, L=LKICK, KICK:=KCV02;
MCH01: HKICKER, L=LKICK, KICK:=KCH01;
MC001: KICKER, L=0.1, VKICK:=KXV001, HKICK:=KXH001;

The class VKICKER or HKICKER refer to orbit correctors for the vertical and horizontal planes respectively. The attribute \textit{KICK} refers to the corresponding plane only. The single class KICKER can be used to specify orbit correctors for both planes. In that case, two attributes \textit{HKICK} and \textit{VKICK} are needed to separate the functions in the two planes.

In the above example, the correctors and their strengths are given individual names which allows to set them explicitly to independent values. For the standard orbit correction with MAD-X this is however not always necessary. The declarations of the kicks as deferred expressions allows the kicks to be changed explicitly or by the program.

3.4 Multipoles

A special class of elements is defined with the keyword MULTIPOLE. These are general elements of zero length (thin lenses) and can be used with one or more components of any order. All thin elements can be written as multipoles in the form:

MPM: MULTIPOLE;
MPL01: MPM, LRAD=0.0, TILT=angle,
KNL\{k_{n0}L, k_{n1}L, k_{n2}L, k_{n3}L, \ldots \},
KSL\{k_{s0}L, k_{s1}L, k_{s2}L, k_{s3}L, \ldots \};

The components KNL and KSL are the normal and skew components of the multipole multiplied by the relevant magnetic length. Note that the strength definitions are position dependent, therefore leading zeroes must be filled for components that do not exist. The attribute LRAD is a fictitious length, which is only used to compute synchrotron radiation effects. For the computation of lattice functions etc., it can be set to some dummy value.

Using multipoles, a thin quadrupole can be defined as:

QFT: MPM, LRAD=0, KNL\{0, k_{n1}L, 0, 0 \};

The thin lens version of a dipole would be written as:

MBT: MPM, LRAD=0, KNL\{k_{n0}L, 0, 0, 0 \};

3.5 Markers

The element class \textit{MARKER} is used to insert an inactive element at a position \textit{s} for later use, e.g. as a reference. The syntax is:

START_IP: MARKER, AT = 1839.872;
If present in a sequence, the lattice functions are calculated at their positions and they play an important role for matching.

A complete list of keywords and pre-defined element classes is found in the reference manual [1, 2].

### 3.6 Element position in a SEQUENCE

The representation of the machine is called a *sequence*. It defines the order in which the elements appear in the accelerator or beam line. In a simple case, a sequence can be defined like:

```plaintext
seq_name: SEQUENCE, REFER=CENTRE, LENGTH=6912.00;
...
MQF05: MQL, AT = 256.0000;
BPMH05: MONITOR, AT = 1.75, FROM=MQF05;
MCH05: HKICKER, AT = 2.10, FROM=MQF05;
MBL05.002: MBL, AT = 265.9000;
MBL05.002: MBL, AT = 278.1000;
MQD05: MQL, AT = 288.0000;
BPMV05: MONITOR, AT = 1.75, FROM=MQD05;
MCV05: VKICKER, AT = 2.10, FROM=MQD05;
MBL05.003: MBL, AT = 297.9000;
MBL05.004: MBL, AT = 310.1000;
...
ENDSEQUENCE;
```

The keywords `SEQUENCE` and `ENDSEQUENCE` define the beginning and end of the definition and the sequence is assigned a name `seq_name`.

The statements look familiar and the additional attribute `AT` defines the position relative to the beginning of the sequence. A position relative to an existing element can be assigned with the `FROM` attribute. The total `LENGTH` of the sequence is specified on the header line of the sequence. The positions can be defined at the CENTRE, ENTRY or EXIT of an element, indicated by the `REFER` attribute. The names given to the elements must be unique, i.e. must not appear twice in the same sequence.

Several sequences with different names can be defined in the same file.

In the example above we have assigned the position to named elements. A second possibility is to use class names like in:

```plaintext
...
MBL: MBL, AT = 278.1000;
MQD: MQD, AT = 288.0000;
BPM: BPM, AT = 1.75, FROM=MQD05;
MCV: MCV, AT = 2.10, FROM=MQD05;
MBL: MBL, AT = 297.9000;
MBL: MBL, AT = 310.1000;
...
assuming BPM, MCV etc. have been defined as classes before. In this case all elements have the same name which is the name of the class and they cannot be distinguished by name.

Finally, a previously defined sequence can be inserted, allowing the possibility to nest sequences. In the example 1 in the appendix I have defined a complete machine using the commands already discussed up to now. The dipoles are defined as thin elements whereas the quadrupoles and sextupoles have a finite length.

7
3.7 Using repetitive definition for periodic machines

The sequence of a periodic machine or the periodic part of a machine can be defined using the MAD-X macro commands. After the usual definition of the cell length \( l_{\text{cell}} \), the half length of a quadrupole \( l_{\text{quad}} \) and the number of cells \( n_{\text{cell}} \), the whole machine can be defined with a \texttt{while}-loop:

\[
\begin{align*}
n &= 1; \\
\text{while} \ (n < n_{\text{cell}}+1) \{} \\
\text{qf: qf, at} &= (n-1) \ast l_{\text{cell}} + l_{\text{quad}}; \\
\text{lsf: lsf, at} &= (n-1) \ast l_{\text{cell}} + l_{\text{quad}} + 2.5; \\
\text{ch: ch, at} &= (n-1) \ast l_{\text{cell}} + l_{\text{quad}} + 3.1; \\
\text{bpm: bpm, at} &= (n-1) \ast l_{\text{cell}} + l_{\text{quad}} + 3.2; \\
\text{mbsps: mbsps, at} &= (n-1) \ast l_{\text{cell}} + l_{\text{quad}} + 3.5; \\
\text{mbsps: mbsps, at} &= (n-1) \ast l_{\text{cell}} + l_{\text{quad}} + 9.9; \\
\text{mbsps: mbsps, at} &= (n-1) \ast l_{\text{cell}} + l_{\text{quad}} + 22.1; \\
\text{mbsps: mbsps, at} &= (n-1) \ast l_{\text{cell}} + l_{\text{quad}} + 28.5; \\
\text{qd: qd, at} &= (n-1) \ast l_{\text{cell}} + l_{\text{quad}} + 32.0; \\
\text{lsd: lsd, at} &= (n-1) \ast l_{\text{cell}} + l_{\text{quad}} + 34.5; \\
\text{cv: cv, at} &= (n-1) \ast l_{\text{cell}} + l_{\text{quad}} + 35.1; \\
\text{bpm: bpm, at} &= (n-1) \ast l_{\text{cell}} + l_{\text{quad}} + 35.2; \\
\text{mbsps: mbsps, at} &= (n-1) \ast l_{\text{cell}} + l_{\text{quad}} + 35.5; \\
\text{mbsps: mbsps, at} &= (n-1) \ast l_{\text{cell}} + l_{\text{quad}} + 41.9; \\
\text{mbsps: mbsps, at} &= (n-1) \ast l_{\text{cell}} + l_{\text{quad}} + 54.1; \\
\text{mbsps: mbsps, at} &= (n-1) \ast l_{\text{cell}} + l_{\text{quad}} + 60.5; \\
\text{n} &= \text{n + 1}; \\
\text{\}}
\end{align*}
\]

The two types of sequence definitions are entirely equivalent.

A complete definition using this technique is given in example 2 in the appendix 1.2. The MAD-X commands and the resulting output are identical to example 1.

4 MAD-X commands

In addition to the statements which are used to define a machine, the MAD-X commands are used to define and execute actions on the machines, e.g. calculations of Twiss functions, I/O of the lattices, particle tracking etc. An important part of the design procedure is lattice matching, i.e. to vary element parameters to make machine properties (e.g. Twiss functions) assuming defined values at specified positions (e.g. interaction points etc.). A complete description of all MAD-X commands is found in [1, 2]. Here I shall list the most important commands which are necessary to do the basic calculations.

4.1 BEAM command

Some of the MAD-X actions require the knowledge of the beam properties. They are defined with the \texttt{BEAM} command:

\texttt{BEAM, PARTICLE=\texttt{name}, ENERGY=\texttt{xxx}, SEQUENCE=\texttt{sname};}

The name of the particle type can be given as well as the particle’s energy. The properties (e.g. mass and charge) of the most important particles are known to MAD-X. Alternatively, the mass and charge can be specified with \texttt{MASS =} and \texttt{CHARGE =}. When the \texttt{SEQUENCE} attribute is given, it will assign the
beam only to this particular sequence, otherwise to the active sequence. A complete list of all possible beam quantities is found in [1, 2].

4.2 Input definition

MAD-X statements and commands can be given on the standard input or can be read from a file with:

CALL,FILE="filename";

This file can contain one or more sequences, part of a sequence or commands and is inserted at the position of the call.

After a sequence has been read, it can be used with:

USE, PERIOD=sname;

This command will expand the specified sequence, insert the drift spaces and make it active.

4.3 MAD-X actions

MAD-X actions are executed to perform operations on the available machines. To calculate the linear lattice functions (Twiss parameters) around the machine, the action:

TWISS;

must be executed, which operates on the sequence defined in the last USE command. However, a sequence can be specified explicitly on the Twiss command. A summary table is given after execution.

4.4 MAD-X output

The TWISS command can be modified to specify the wanted output:

SELECT,FLAG=TWISS,COLUMN=NAME,S,MUX,BETX,MUY,BETY;
TWISS,FILE="twiss.out";

In the SELECT command the lattice functions wanted can be specified before TWISS is executed. The full list of the lattice functions is given in [1, 2]. The lattice functions are written into the file "twiss.out".

The SELECT command can be used to restrict the output to only a range or type of elements:

SELECT,FLAG=TWISS,RANGE=beg/end;
or: SELECT,FLAG=TWISS,PATTERN="Q.*";

The first will output the lattice functions only within the specified range and the second would restrict the output to all elements starting with the specified pattern in the element name. The SELECT commands act for the desired action (FLAG=) and can be accumulated or overwritten.

4.5 MAD-X graphical output

MAD-X has a builtin graphics package. To plot lattice functions for example, the command sequence:

SELECT,FLAG=TWISS;
TWISS,FILE="twiss.beta";
PLOT, HAXIS=S, VAXIS=BETX, BETY;
may be used to plot the horizontal and vertical $\beta$-functions as a function of the position $s$. The RANGE attribute can be used with the PLOT command. An output file and a PS-file are written simultaneously. For details and all options see [1, 2].

### 4.6 MAD-X example

In the second part of example 1 the necessary MAD-X commands are given to calculated the lattice functions with the TWISS command, write them onto a file and plot them in postscript format. At the end of the execution of a TWISS command, a summary table is printed:

```
++++++ table: summ

<table>
<thead>
<tr>
<th>length</th>
<th>orbit5</th>
<th>alfa</th>
<th>gammatr</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.912000e+03</td>
<td>-0.000000e+00</td>
<td>1.6807003e-03</td>
<td>2.4392418e+01</td>
</tr>
</tbody>
</table>
```

```
q1          | dq1    | betx     | dxmax   |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2.6580000e+01</td>
<td>-3.3561557e+01</td>
<td>1.0754431e+02</td>
<td>2.5680113e+00</td>
</tr>
</tbody>
</table>
```

```
dxrms | xcomax | xcorms | q2 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.9304378e+00</td>
<td>0.00000000e+00</td>
<td>0.00000000e+00</td>
<td>2.66200000e+01</td>
</tr>
</tbody>
</table>
```

```
dq2        | betymax | dymax   | dyrms |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-3.3598479e+01</td>
<td>1.0749730e+02</td>
<td>0.00000000e+00</td>
<td>0.00000000e+00</td>
</tr>
</tbody>
</table>
```

```
ycomax | ycorms | deltap |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00000000e+00</td>
<td>0.00000000e+00</td>
<td>0.00000000e+00</td>
</tr>
</tbody>
</table>
```

The main parameters of the lattice are summarized in this table, such as horizontal and vertical tunes (Q1, Q2), chromaticities (DQ1, DQ2), etc.

The lattice functions $\beta_x$ and $D_x$ are plotted by the above command sequence as shown in Fig.2. The functions are plotted between the 10th and 16th quadrupole of the class QD as specified in the RANGE attribute.

As requested, the lattice functions are written to a file "twiss.out" and its format is shown in the last part of appendix 1. At the beginning of this file the basic parameters are summarized again.
Figure 2: Lattice functions computed and plotted by MAD-X.
5 Matching with MAD-X

The adjustment of machine properties, i.e. matching, is a vital part of the design process and a detail description is beyond the scope of this introduction. However some very basic features will be demonstrated by some examples. A basic tutorial on some matching techniques is found in [4, 5].

5.1 Global matching

Some global machine parameters such as tune or chromaticity can be adjusted by global matching. The following two example are used as a demonstration:

MATCH, SEQUENCE=CASSPS;
VARY,NAME=KQF, STEP=0.00001;
VARY,NAME=KQD, STEP=0.00001;
GLOBAL, SEQUENCE=CASSPS, Q1=26.58;
GLOBAL, SEQUENCE=CASSPS, Q2=26.62;
LMDIF, CALLS=10, TOLERANCE=1.0E-21;
ENDMATCH;

MATCH, SEQUENCE=CASSPS;
VARY,NAME=KSF, STEP=0.00001;
VARY,NAME=KSD, STEP=0.00001;
GLOBAL, SEQUENCE=CASSPS, DQ1=0.0;
GLOBAL, SEQUENCE=CASSPS, DQ2=0.0;
LMDIF, CALLS=10, TOLERANCE=1.0E-21;
ENDMATCH;

The matching attributes and commands are enclosed between the match and endmatch statements. The desired sequence can be specified. In the first example the global horizontal and vertical tunes are matched to the desired values. The strengths of the main quadrupoles (kqf and kqd) are varied in the procedure. In the second example the global chromaticities are matched to zero, by variation of the sextupole strengths ksf and ksd. Other attributes control the method used and the quality of the procedure. The new values of the strengths are now associated with the sequence. The following execution of the TWISS command would therefore produce the new parameters.

NOTE: the latter is only true when the quadrupole strengths are defined using deferred expressions, i.e:

QF: QUADRUPOLE, K1:=KQF;

Otherwise the new strengths KQF and KQD are calculated all right, but not assigned to the elements. I.e. they will not be used in subsequent calculations, e.g. computing lattice functions with the TWISS command.

5.2 Local and insertion matching

Probably the most important matching procedures are those which are used to modify the lattice locally, e.g. for inserting non-periodic regions for experiments, collimation systems etc. In order to avoid a distortion of other parts of the machine, the matching must be restricted to the local region and additional constraints must ensure the smooth continuation into the periodic part of the machine.

The example below is a simple matching of a symmetric low β-insertion using four independent quadrupoles. The matching is restricted to the range between the elements left and right where the normal lattice parameters are given as constraints. For more details on this example see [5].
MATCH, SEQUENCE=CASCELL5, RANGE=LEFT/RIGHT, BETX=28.2, BETY=87.0;
VARY, NAME=KQ1.L, STEP=0.00001;
VARY, NAME=KQ2.L, STEP=0.00001;
VARY, NAME=KQ3.L, STEP=0.00001;
VARY, NAME=KQ4.L, STEP=0.00001;
CONSTRAINT, RANGE=RIGHT, SEQUENCE=CASCELL5, BETX=28.2, BETY=87.0;
CONSTRAINT, RANGE=IP, SEQUENCE=CASCELL5, BETX=10.0, BETY=1.0;
LMDIF, CALLS=100, TOLERANCE=1.0E-21;
ENDMATCH;

5.3 Example: matching a vertical orbit bump

As an example I show the commands to match a vertical orbit bump of 5 mm at a monitor. Three vertical orbit correctors are used to create and close the bump.

// Match a vertical orbit bump of 5 mm at monitor BPMV8
MATCH, ORBIT;
CONSTRAINT, RANGE=BPMV5, Y=0.0, PY=0.0;  // make sure bump is closed
CONSTRAINT, RANGE=BPMV8, Y=0.005;        // bump is 5 mm vertical
CONSTRAINT, RANGE=BPMV11, Y=0.0, PY=0.0;  // make sure bump is closed
VARY, NAME=KCV7, STEP=0.0001;
VARY, NAME=KCV9, STEP=0.0001;
VARY, NAME=KCV10, STEP=0.0001;
LMDIF, CALLS=100, TOLERANCE=1.0E-20;
ENDMATCH;
The complete example can be found as example 3 in the appendix. It uses the sequence as defined in example 4 (macro definition, see later). The resulting vertical orbit is shown in Fig.3.

Figure 3: Vertical orbit bump as example for local matching.
6 Error definition

During the design process of a machine it becomes important to test it against imperfections. For that purpose, alignment and field errors can be assigned to all machine elements. The calculations will take these imperfections into account and correction procedures (e.g. orbit correction) are available in MAD-X to test possible correction strategies.

6.1 Alignment errors

The elements in a machine can be misaligned with the available MAD-X error actions. The command sequence:

SELECT, FLAG=ERROR, CLASS=MQ;
EALIGN, DX:=GAUSS()*0.0005; DY:=GAUSS()*0.0002;

assigns alignment errors to all quadrupoles belonging to the class MQ with a r.m.s. value of 0.5 mm in the horizontal and 0.2 mm in the vertical plane, both following a Gaussian distribution. In that case again the use of deferred expressions is of utmost importance. To assign the errors, the program steps through the sequence and for every element of the selected class the corresponding misalignments DX and DY are evaluated each time. Using the standard expression, the misalignments DX and DY are calculated once and all elements of the selected class get the same error. For a complete list of all misalignment options see [1, 2].

6.2 Field errors

The program allows to assign field errors of any order to the machine elements with commands like:

SELECT, FLAG=ERROR, CLASS=MB;
EFCOMP, RADIUS:=0.017, ORDER:=0,
DKNR:=[0,0,GAUSS()*7E-4,GAUSS()*1E-4,0,0],
DKSR:=[0,0,GAUSS()*3E-4,GAUSS()*6E-4,0,0],

In this example normal and skew field errors (sextupole and octupole) are assigned to dipole magnets of the class MB. It is possible to assign absolute or relative field errors, the latter normalized to the strength of the corresponding element. The RADIUS (reference for the measurement) and ORDER control this behaviour. For a detailed discussion see [1, 2].

7 Orbit correction

A misaligned machine can be corrected using the MAD-X orbit correction procedures [6]. The input data is taken from the last TWISS table, i.e. TWISS must run before a correction can be executed.

Very basic closed orbit correction statements are of the form:

CORRECT, PLANE=X, NCORR=20, ERROR=1.0E-04;

or

CORRECT, MODE=SVD;

For all details and options see [1, 2, 6].
8 Advanced options and commands

MAD-X has many more features and commands for advanced design and evaluation of accelerator lattices. Most prominent are the evaluation of beam parameters (in case of radiation), geometrical survey, tracking and physical and dynamic aperture determination. However, a full description is well beyond the scope of this simplified introduction. To get a flavor, I shall give two examples, one for a simple tracking and another for the advanced use of macros.

8.1 Particle tracking

The example shown below demonstrates particle tracking in MAD-X. It shows the simultaneous tracking of 20 particles in horizontal phase space where the particles are distributed on a circle in x-px phase space. All commands are enclosed between the keywords \texttt{track} and \texttt{endtrack}. The initial coordinates of the particles are assigned with the command \texttt{start} and the tracking is executed with \texttt{run}.

Tracking in MAD-X is possible using thin lenses. A lattice defined with thick elements has to be converted to thin lenses with the command \texttt{MAKETHIN} before the tracking can be done. For details on the command \texttt{MAKETHIN} consult the reference manual [1, 2].

\begin{verbatim}
MAKETHIN,SEQUENCE=CASSPS;
USE,SEQUENCE=CASSPS;
TRACK;
NSTEP = 20;
RAD = 100.0E-06;
ANGSTP = 2*PI/NSTEP;

N = 0;
WHILE(N <= NSTEP) {
  ANG = N*ANGSTP;
  XS = RAD*COS(ANG);
  XPS = RAD*SIN(ANG);
  VALUE, XS,XPS;
  START,X=XS,PX=XPS;
  N = N + 1;
}
RUN,TURNS=1024;
ENDTRACK;
STOP;
\end{verbatim}

The use of tracking may require additional attributes in the BEAM command, e.g. in the case of beam-beam elements required in the simulation. The above example shows the power of the input language.

8.2 Particle tracking with PTC

Tracking with PTC allows the use of thick elements. An example how to invoke PTC tracking and to produce the output is shown below.

\begin{verbatim}
ptc_create_universe;
ptc_create_layout,model=2,method=6,nst=10,exact;

ptc_start, x= 3e-2, px=0, y= 3e-2, py=0;
ptc_start, x= 6e-2, px=0, y= 6e-2, py=0;
ptc_start, x= 9e-2, px=0, y= 9e-2, py=0;
\end{verbatim}
8.3 Use of macros

The power of the MAD-X input language is further enhanced by the use of *macros*. For illustration, example 2 in the appendix has been modified. For some cases, it is required that the elements have distinct names, (e.g. where all elements must be treated as separate objects, such as orbit corrections). This can be easily done by editing example 1, where every element is listed on a separate line. Editing the example 2 with the *while* loop would fail because two elements must not have the same name. Using the macro language, the while loop can be modified like in example 3 in appendix 1.3. The definition of a cell is now done within the subroutine *inst*. This subroutine takes input parameters from the calling MAD-input and most important, can change the names of the elements, using the input information. The result of this scheme is the same as before, however the orbit correctors, their corresponding strength parameters and the beam position monitors are now numbered sequentially. A MAD-X input file and the corresponding Twiss output are shown in the second part of example 3. The increasing sequence numbers as part of the element names are now clearly visible. In this example the quadrupoles have been misaligned in the two planes following a Gaussian distribution with 0.1 mm and 0.2 mm r.m.s. respectively.

Therefore the horizontal and vertical orbit is distorted and the maximum and r.m.s. values can be found in the Twiss summary table.

9 How to run MAD-X?

MAD-X can be run either interactively or in batch mode.

9.1 Interactive mode

To run MAD-X interactively, one can execute MAD-X and input the commands and statements in the command line of the standard input.

Alternatively, one or more files with commands and statements can be read using the command: 

```
CALL, FILE="filename";
```
9.2 Batch mode

MAD-X can be run as a background of batch program by redirecting an input file into the MAD-X standard input. For UNIX (LINUX) like:

```
madx < inputfile
```

The output is normally send to the standard output, unless it is redirected.
In the following, I list a few examples which may be referenced in the text. For all cases the sequence definition is found in the first part and a MAD-X input file to use the sequence is given in the second part of each example. All examples can be found on the web in [7].

10.1 APPENDIX 1.1: Example 1, Simplest case

10.1.1 Sequence definition (spsall.seq)

// define the total length
circum=6912.00;

// define number of cells and therefore cell length
ncell = 108;
lcell = circum/ncell;

// define lengths of elements and half lengths
lquad = 3.085;
lmb = 6.260;
lsex = 1.0;

// forces and other constants;
// element definitions;

// define bending magnet as multipole
mbps: multipole, lrad=dummy, knl={2.0*pi/(8*ncell)};

// define quadrupole and their strengths
qsps: quadrupole, l=lquad;
qf: qsps, k1:=kqf;
qd: qsps, k1:=kqd;
kqf = 1.4631475E-02;
kqd = -1.4643443E-02;

// define sextupoles for chromaticity correction
lsf: sextupole, l=lsex,k2:=ksf;
lsd: sextupole, l=lsex,k2:=ksd;
ksf = 2.0284442E-02;
ksd = -3.8394267E-02;

// define orbit correctors and beam position monitors
bpm: monitor, l=0.1;
ch: hkicker, l=0.1;
cv: vkicker, l=0.1;

cassps: sequence, refer=centre, l = circum;
start_machine: marker, at = 0;
qf, at = 1.5425;
lsf, at = 4.0425;
ch, at = 4.6425;
bpm, at = 4.7425;
mbps, at = 5.0425;
mbps, at = 11.4425;
mbps, at = 23.6425;
mbps, at = 30.0425;
qd, at = 33.5425;
lsd, at = 36.0425;
cv, at = 36.6425;
bpm, at = 36.7425;
mbps, at = 37.0425;
mbps, at = 43.4425;
mbps, at = 55.6425;
mbps, at = 62.0425;
qf, at = 65.5425;
lsf, at = 68.0425;
ch, at = 68.6425;
bpm, at = 68.7425;
mbps, at = 69.0425;
mbps, at = 75.4425;
mbps, at = 87.6425;
mbps, at = 94.0425;
qd, at = 97.5425;
lsd, at = 100.0425;
cv, at = 100.6425;
bpm, at = 100.7425;
mbps, at = 101.0425;
mbps, at = 107.4425;
mbps, at = 119.6425;
mbps, at = 126.0425;
qf, at = 129.5425;
lsf, at = 132.0425;
ch, at = 132.6425;
bpm, at = 132.7425;
mbps, at = 133.0425;
mbps, at = 139.4425;
mbps, at = 151.6425;
mbps, at = 158.0425;
qd, at = 161.5425;
lsd, at = 164.0425;
cv, at = 164.6425;
bpm, at = 164.7425;
mbps, at = 165.0425;
mbps, at = 171.4425;
...
...
...
mbps, at = 6775.6425;
mbps, at = 6782.0425;
qf, at = 6785.5425;
lsf, at = 6788.0425;
10.1.2 MAD-X directives

TITLE, s='MAD-X test';

// Read input file with machine description
call file="spsall.seq";
option,-echo;

// Define the beam for the machine
Beam, particle = proton, sequence=cassps, energy = 450.0;

// Use the sequence with the name: cassps
use, period=cassps;

// Define the type and amount of output for the action TWISS
select, flag=twiss, column=name,s,x,y,mux,betx,muy,bety,dx,dy;
// Execute the Twiss command to calculate the Twiss parameters
// Compute at the centres of the elements and write to: twiss.out
twiss, centre, file=twiss.out;

// Plot the horizontal and vertical beta function between the
// 10th and 16th occurrence of a defocussing quadrupole
plot, haxis=s, vaxis=x, betx, bety, colour=100,
    range=qd[10]/qd[16];
plot, haxis=s, vaxis=dx, colour=100,
    range=qd[10]/qd[36];
stop;

10.1.3 TWISS summary table

++++++ table: summ

<table>
<thead>
<tr>
<th>length</th>
<th>orbit5</th>
<th>alfa</th>
<th>gammatr</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.9120000e+03</td>
<td>-0.0000000e+00</td>
<td>1.6807003e-03</td>
<td>2.4392418e+01</td>
</tr>
<tr>
<td>q1</td>
<td>dq1</td>
<td>betxmax</td>
<td>dxmax</td>
</tr>
<tr>
<td>2.6580000e+01</td>
<td>-3.3561557e+01</td>
<td>1.0754431e+02</td>
<td>2.5680113e+00</td>
</tr>
<tr>
<td>dxrms</td>
<td>xcomax</td>
<td>xcorms</td>
<td>q2</td>
</tr>
<tr>
<td>1.9304378e+00</td>
<td>0.0000000e+00</td>
<td>0.0000000e+00</td>
<td>2.6620000e+01</td>
</tr>
<tr>
<td>dq2</td>
<td>betymax</td>
<td>dymax</td>
<td>dyrms</td>
</tr>
<tr>
<td>-3.3598479e+01</td>
<td>1.0749730e+02</td>
<td>0.0000000e+00</td>
<td>0.0000000e+00</td>
</tr>
<tr>
<td>ycomax</td>
<td>ycorms</td>
<td>deltapi</td>
<td></td>
</tr>
<tr>
<td>0.0000000e+00</td>
<td>0.0000000e+00</td>
<td>0.0000000e+00</td>
<td></td>
</tr>
</tbody>
</table>
10.1.4  TWISS lattice functions written to the file "twiss.out"

@ NAME %05s "TWISS"
@ TYPE %05s "TWISS"
@ SEQUENCE %06s "CASSPS"
@ PARTICLE %06s "PROTON"
@ MASS %le 0.938271998
@ CHARGE %le 1
@ ENERGY %le 450
@ PC %le 449.999021827
@ GAMMA %le 479.605062241
@ KBUNCH %le 1
@ BCURRENT %le 0
@ SIGE %le 0
@ SIGT %le 0
@ NPART %le 0
@ EX %le 1
@ EY %le 1
@ ET %le 1
@ LENGTH %le 6912
@ ALFA %le 0.00168070032886
@ ORBIT5 %le -0
@ GAMMATR %le 24.3924182122
@ Q1 %le 26.58
@ Q2 %le 26.62
@ DQ1 %le -33.5615573373
@ DQ2 %le -33.5984799903
@ DXMAX %le 2.568011301
@ DYMAX %le 0
@ XCOMAX %le 0
@ YCOMAX %le 0
@ BETXMAX %le 107.544319159
@ BETYMAX %le 107.497305443
@ XCORMS %le 0
@ YCORMS %le 0
@ DXRMS %le 1.93043782638
@ DYRMS %le 0
@ DELTAP %le 0
@ TITLE %01s "s"
@ ORIGIN %16s "MAD-X 1.11 Linux"
@ DATE %08s "23/10/03"
@ TIME %08s "11.36.14"

* NAME  S   BETX   DX
$ %s  %le  %le  %le
"CASSPS$START" 0 103.8655173 2.523441048
"START_MACHINE" 0 103.8655173 2.523441048
"QF" 1.5425 107.5443192 2.568011301
"DRIFT_0" 3.11375 103.7300292 2.521784419
"LSF" 3.6425 101.2568359 2.491316849
"DRIFT_1" 4.5925 96.90195183 2.43657606

22
10.2.1 Sequence definition (sps.seq)

// define the total length
circum=6912.0;

// define number of cells and therefore cell length
ncell = 108;
lcell = circum/ncell;

// define lengths of elements and half lengths
lquad = 3.085;
lquad2 = lquad/2.;
lsex = 1.0;

// forces and other constants;
// element definitions;

// define bending magnet as multipole
mbbps: multipole, lrad=dummy, knl={2.0*pi/(8*ncell)};

// define quadrupole and their strengths
qqsps: quadrupole, l=lquad;
qf: qqsps, k1:=kqf;
qd: qqsps, k1:=kd;
kqf = 1.4631475E-02;
kqd = -1.4643443E-02;

// define sextupoles for chromaticity correction
lsf: sextupole, l=lsex,k2:=ksf;
lsd: sextupole, l=lsex,k2:=ksd;
ksf = 2.0284442E-02;
ksd = -3.8394267E-02;

// define orbit correctors and beam position monitors
bpm: monitor, l=0.1;
ch: hkicker, l=0.1;
cv: vkicker, l=0.1;

// sequence declaration;
cassps: sequence, refer=centre, l:=circum;
start_machine: marker, at = 0;

// This defines ONE cell, repeat NCELL times
// to get the full machine
// SPS has 8 bending magnets per cell
n = 1;
while (n < ncell+1) {
  qf: qf, at=(n-1)*lcell+lquad2;
  lsf: lsf, at=(n-1)*lcell+lquad2+2.5;
10.2.2 MAD-X directives

TITLE, s='MAD-X test';

// Read input file with machine description
call file="sps.seq";
option,-echo;

// Define the beam for the machine
Beam, particle = proton, sequence=cassps, energy = 450.0;

// Use the sequence with the name: cassps
use, period=cassps;

// Define the type and amount of output for the action TWISS
select,flag=twiss,column=name,s,x,y,mux,betx,muy,bety,dx,dy;

// Execute the Twiss command to calculate the Twiss parameters
// Compute at the centres of the elements and write to: twiss.out
twiss,centre,file=twiss.out;

// Plot the horizontal and vertical beta function between the
// 10th and 16th occurrence of a defocussing quadrupole
plot, haxis=s, vaxis=x, betx, bety,colour=100,
range=qd[10]/qd[16];
plot, haxis=s, vaxis=dx, colour=100,
range=qd[10]/qd[36];

stop;
10.3 APPENDIX 1.3: Example 3, Matching a local orbit bump

10.3.1 MAD-X directives

// Read input file with machine description
// This machine is constructed with macro
// subroutine INST()
// Match a vertical orbit bump at monitor BPMV8
call file="spsmac.seq";
option,-echo;

// Define the beam for the machine
Beam, particle = proton, sequence=cassps, energy = 450.0;

// Use the sequence with the name: cassps
use, sequence=cassps;

// Match a vertical orbit bump of 5 mm at monitor BPMV8
match,orbit;
constraint,range=bpmv5,y=0.0,py=0.0;
constraint,range=bpmv8,y=0.005;
constraint,range=bpmv11,y=0.0,py=0.0;
vary,name=kcv7,step=0.0001;
vary,name=kcv9,step=0.0001;
vary,name=kcv10,step=0.0001;
lmdif,calls=100,tolerance=1.0E-20;
endmatch;

// Define the type and amount of output
select,flag=twiss,class=monitor,column=name,s,x,px,y,py;
select,flag=twiss,class=hkicker,column=name,s,x,px,y,py;
select,flag=twiss,class=vkicker,column=name,s,x,px,y,py;

// Execute the Twiss command to calculate the Twiss parameters
// Compute at the centre of the element and write to: twiss.out
twiss,save,centre,file=twiss.out;

// Plot the vertical orbit between the
// 1st and 20th beam position monitor
plot, haxis=s, vaxis=y,colour=100,
    vmin=-0.001,vmax=0.011,
    range=bpmv3/bpmv13;
stop;
10.4.1 Sequence definition (spsmac.seq)

// define a subroutine "inst" to insert elements
// with numbering
// all strings "nx" in the macro are replaced by
// the input value of nx.
inst(nx,n,lcell,lquad2): macro = {
  qf: qf, at=(n-1)*lcell+lquad2;
  lsf: lsf, at=(n-1)*lcell+lquad2+2.5;
  chnx: hkicker,l=0.0,kick:=kchnx,
         at=(n-1)*lcell+lquad2+3.1;
  bpmhnx: monitor,l=0.0,
          at=(n-1)*lcell+lquad2+3.2;
  mbsps: mbsps, at=(n-1)*lcell+lquad2+3.50;
  mbsps: mbsps, at=(n-1)*lcell+lquad2+9.90;
  mbsps: mbsps, at=(n-1)*lcell+lquad2+22.10;
  mbsps: mbsps, at=(n-1)*lcell+lquad2+28.50;
  qd: qd, at=(n-1)*lcell+lquad2+32.00;
  lsd: lsd, at=(n-1)*lcell+lquad2+34.50;
  cvnx: vkicker,l=0.0,kick:=kcvnx,
         at=(n-1)*lcell+lquad2+35.10;
  bpmvnx: monitor,l=0.0,
          at=(n-1)*lcell+lquad2+35.20;
  mbsps: mbsps, at=(n-1)*lcell+lquad2+35.50;
  mbsps: mbsps, at=(n-1)*lcell+lquad2+41.90;
  mbsps: mbsps, at=(n-1)*lcell+lquad2+54.10;
  mbsps: mbsps, at=(n-1)*lcell+lquad2+60.50;

  n = n + 1;
}

// define the total length
circum=6912.0;

// define number of cells and therefore cell length
ncell = 108;
lcell = circum/ncell;

// define lengths of elements and half lengths
lquad = 3.085;
lquad2 = lquad/2.;
lquad3 = 0.0;
lmb = 6.260;
lmb2 = lmb/2.;
lsex = 1.0;

// forces and other constants;
// element definitions;
// define bending magnet as multipole
mbps: multipole, lrad=dummy, l=lmb, knl={2.0*pi/(8*ncell)};

// define quadrupole and their strengths
qsp: quadrupole, l=lquad;
qf: qsp, k1:=kqf;
qd: qsp, k1:=kqd;
kqf = 1.4631475E-02;
kqd = -1.4643443E-02;

// define sextupoles for chromaticity correction
lsf: sextupole, l=lsex,k2:=ksf;
lsd: sextupole, l=lsex,k2:=ksd;
ksf = 2.0284442E-02;
ksd = -3.8394267E-02;

// define orbit correctors and beam position monitors
bpm: monitor, l=0.1;
ch: hkicker, l=0.1;
cv: vkicker, l=0.1;

// sequence declaration;
cassps: sequence, refer=centre, l=circum;
start_machine: marker, at = 0;

// This defines ONE cell, repeat NCELL times
// to get the full machine
// SPS has 8 bending magnets per cell
n = 1;
while (n < ncell+1) {
   // here we call the macro, cell number n is argument
   // and used for numbering the elements
   exec inst(n,n,lcell,lquad2);
}
end_machine: marker at=circum;
endsequence;

10.4.2 MAD-X directives

TITLE, s='MAD-X test';

// Read input file with machine description
// This machine is constructed with macro
// subroutine INST()
call file="spsmac.seq";
option,-echo;

// Define the beam for the machine
Beam, particle = proton, sequence=cassps, energy = 450.0;
// Use the sequence with the name: cassps
use, sequence=cassps;

eoption,add=false,seed=62971100;
select,flag=error,pattern="q.*";
ealign,dx:=tgauss(3.0)*1.0e-4,dy:=tgauss(3.0)*2.0e-4;
eprint;

// Define the type and amount of output
select,flag=twiss,class=monitor,column=name,s,x,betx;
select,flag=twiss,class=vkicker,column=name,s,x,betx;
select,flag=twiss,class=hkicker,column=name,s,x,betx;

// Execute the Twiss command to calculate the Twiss parameters
// Compute at the centre of the element and write to: twiss.out
twiss,save,centre,file=twiss.out;

stop;

10.4.3 TWISS summary table

++++++ table: summ

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>length</td>
<td>orbit5</td>
<td>alfa</td>
<td>gammatr</td>
</tr>
<tr>
<td>6.9120000e+03</td>
<td>-0.0000000e+00</td>
<td>1.6804423e-03</td>
<td>2.4394290e+01</td>
</tr>
<tr>
<td>q1</td>
<td>dq1</td>
<td>betxmax</td>
<td>dxmax</td>
</tr>
<tr>
<td>2.6580085e+01</td>
<td>-1.2827690e-04</td>
<td>1.0808509e+02</td>
<td>2.6296100e+00</td>
</tr>
<tr>
<td>dxrms</td>
<td>xcomax</td>
<td>xcorms</td>
<td>q2</td>
</tr>
<tr>
<td>1.9361006e+00</td>
<td>3.1098580e-03</td>
<td>1.0489137e-03</td>
<td>2.6620213e+01</td>
</tr>
<tr>
<td>dq2</td>
<td>betymax</td>
<td>dymax</td>
<td>dyrms</td>
</tr>
<tr>
<td>-5.1039987e-04</td>
<td>1.0787781e+02</td>
<td>4.0349793e-01</td>
<td>1.5327251e-01</td>
</tr>
<tr>
<td>ycomax</td>
<td>ycorms</td>
<td>deltap</td>
<td></td>
</tr>
<tr>
<td>8.6691696e-03</td>
<td>2.8727246e-03</td>
<td>0.0000000e+00</td>
<td></td>
</tr>
</tbody>
</table>
10.4.4  TWISS lattice functions

| NAME | %05s "TWISS" |
| TYPE | %05s "TWISS" |
| SEQUENCE | %06s "CASSPS" |
| PARTICLE | %06s "PROTON" |
| MASS | %le 0.938271998 |
| CHARGE | %le 1 |
| ENERGY | %le 450 |
| PC | %le 449.999021827 |
| GAMMA | %le 479.605062241 |
| KBUNCH | %le 1 |
| BCURRENT | %le 0 |
| SIGE | %le 0 |
| SIGT | %le 0 |
| NPART | %le 0 |
| EX | %le 1 |
| EY | %le 1 |
| ET | %le 1 |
| LENGTH | %le 6912 |
| ALFA | %le 0.00168044235319 |
| ORBIT5 | %le -0 |
| GAMMATR | %le 24.3942904601 |
| Q1 | %le 26.5800856671 |
| Q2 | %le 26.6202187135 |
| DQ1 | %le -0.000128276909332 |
| DQ2 | %le -0.000510399876937 |
| DXMAX | %le 2.629610492 |
| DYMAX | %le 0.403497938523 |
| XCOMAX | %le 0.00310985806321 |
| YCOMAX | %le 0.0086691696294 |
| BETXMAX | %le 108.085099015 |
| BETYMAX | %le 107.877818541 |
| XCORMS | %le 0.00104891378772 |
| YCORMS | %le 0.0028727246142 |
| DXRMS | %le 1.93610068609 |
| DYRMS | %le 0.153272554514 |
| DELTAP | %le 0 |
| TITLE | %01s "s" |
| ORIGIN | %16s "MAD-X 1.11 Linux" |
| DATE | %08s "23/10/03" |
| TIME | %08s "10.54.36" |

* NAME  S  X  BETX

<table>
<thead>
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